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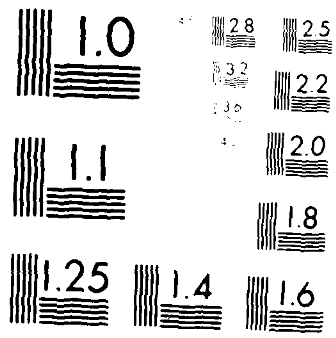
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TECHNICAL REPORT RE-83-5

GROUP COMPLEMENTARY CODES WITH OPTIMIZED APERIODIC CORRELATION

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April 1983

U.S. ARMY MISSILE COMMAND
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This document describes the structure and properties of an innovative new waveform design for pulse compression processing in target sensor systems. The waveform design, which is called group complementary codes, provides sets of binary word groups which have the combined properties of optimized aperiodic autocorrelation and optimized cross correlation between word sets. The optimized aperiodic autocorrelation property allows the implementation of pulse compression processing in sensor systems to achieve zero value temporal or range sidelobes within the principal interpulse period without resorting to

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered) weighting techniques for sidelobe reduction. Weighting techniques for sidelobe reduction in a pulse compression system result in a broadened mainlobe and mismatch loss. These undesirable properties can be avoided through the utilization of group complementary codes.

The orthogonal nature of group complementary codes, apparent from their absence of cross correlation between sets, implies that sensors using these code sets may be deployed in close proximity, using the same carrier frequency without direct path mutual interference when synchronized. Indirect path interference is minimized even when transmissions are not synchronized.

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I. INTRODUCTION

Before group complementary code concepts are described, a short review of the benefits and techniques of pulse compression in sensor systems is presented.[1,2]

Pulse compression involves the rearrangement of the temporal distribution of energy in a pulse in such a way that a long pulse with a given energy is transformed to a shorter pulse with the same energy. The instantaneous power during the shortened pulse is therefore greater than the instantaneous power during the long pulse, since the total energy is the same for both.

Pulse compression is useful in target sensor systems for several reasons. Because target tracking is essentially a process of measurement of the time of arrival of a waveform, pulse compression allows a more precise measurement of arrival time and therefore of the range to the target. If the active sensor is peak power limited, as is usually the case, pulse compression allows long pulse, limited peak power systems to have performance equivalent to a shorter pulse higher peak power system. Of equal or greater importance is the improved range resolution afforded with pulse compression.

Ideally, pulse compression is implemented with matched filters where the processing device is a network with impulse response matched to the time reverse of the long pulse waveform. This matched filter operation results in maximizing the signal-to-noise ratio and in optimum detection of the target. Figure 1 illustrates the concept of pulse compression.

Pulse compression provides a radar sensor with

- IMPROVED RANGE RESOLUTION
- EQUIVALENT DETECTION PERFORMANCE TO A HIGH PEAK POWER SYSTEM
- MAXIMIZED SIGNAL-TO-NOISE AND OPTIMUM TARGET DETECTABILITY THROUGH MATCHED FILTERING

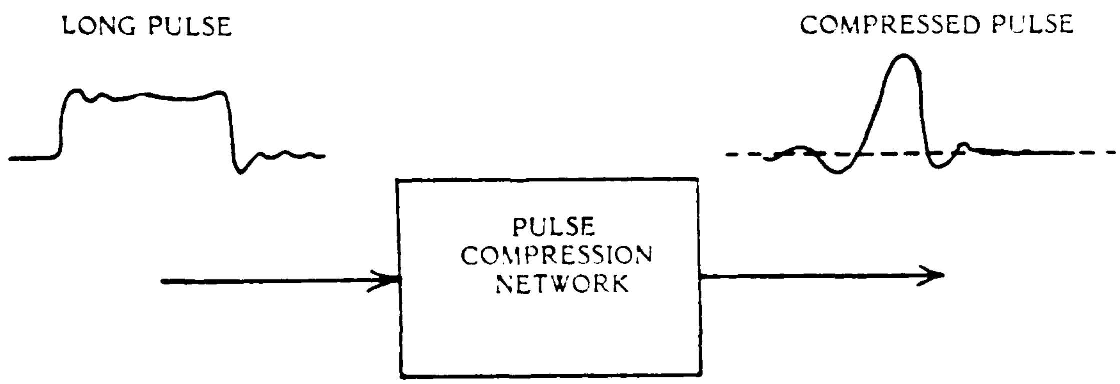


Figure 1. Pulse compression in target sensors.

Pulse compression techniques can be implemented using transversal filtering. Essentially, the method is to delay the energy arriving early in the long pulse period, add it coherently with the energy that arrives later in the pulse, and output the resulting shorter pulse. Two technical issues that arise in the evaluation of the effectiveness of the pulse compression systems are sidelobes in the compressed pulse wave form and the response of the transversed filter to other, nonmatched waveforms which may be present in the received signal. The nonmatched waveforms could be the result of receiving the transmission of other deployed sensors or the result of the transmission of intentional jammers. Figure 2 illustrates the implementation of pulse compression using a matched transversal filter.

An array of waveforms has been used for pulse compression, including binary coding of the phase of a carrier signal (bi-phase modulation using binary codes).[3] Perhaps the best known codes for use in bi-phase modulation implementations of pulse compression are the Barker Codes. Other binary waveforms that have been used for pulse compression include pseudo random codes and random binary codes. Nonbinary waveforms that have been used for pulse compression include FM modulated signals and polyphase codes.

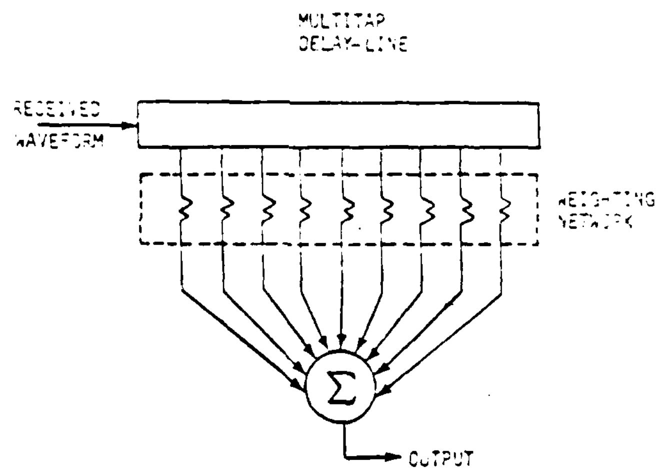


Figure 2. A matched transversal filter for pulse compression, the weighting network provides an inpulse response matched to the time reverse of the input waveform.

A problem that has limited the utility of pulse compression and correlation receivers in radar systems has been the existence of temporal/range sidelobes in the correlation function of the radar waveform. These sidelobes allow out-of-range-gate returns, such as clutter, to compete with a target in a particular range gate. A number of research efforts have addressed this problem in the past, and several waveform designs have resulted in the potential reduction or elimination of the range sidelobe problem. For example, Barker codes (also known as perfect binary words) limit the range sidelobes to a value of 1/N, expressed in the autocorrelation function:

$$|C(\tau)| \leq 1/N, \text{ where } \tau = iT_c \text{ and } i \neq 0,$$

where N is the code length and T_c is the reciprocal of the code rate. Figure 3 illustrates the correlation function of a length 13 Barker Code. Barker codes are known for lengths only up to $N = 13$, and they do not match the desired "perfect" range correlation property,

$$|C(iT)| = \begin{cases} 1 \text{ if } i = 0 \\ 0 \text{ if } i \neq 0 \end{cases} .$$

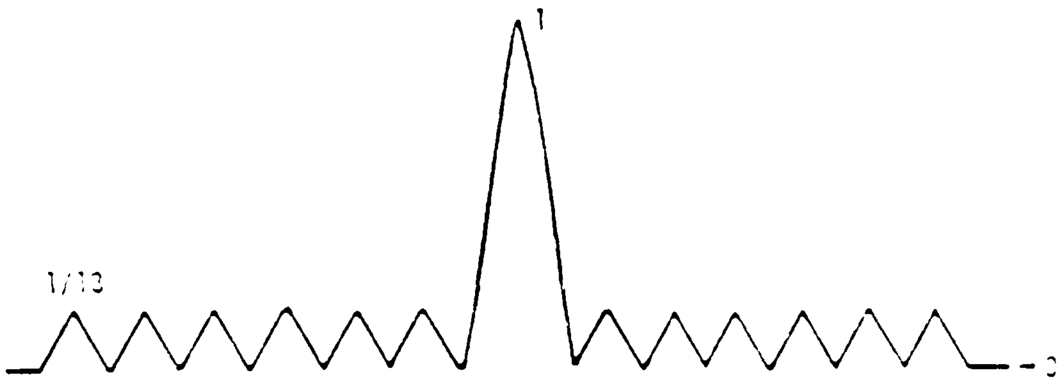


Figure 3. Absolute value of the normalized autocorrelation function of the 13-bit Barker code.

Application of Golay code pairs (also known as complementary sequences) involves processing two coded pulses at a time in a radar processor to eliminated the range sidglobes.[4] These codes have the property that when their individual range sidelobes are combined (algebraic addition), the composite sidelobes completely cancel, yielding the desired perfect correlation property. Complementary sequences are known to exist for a limited number of sequence lengths, including $N = 2, 4, 8, 10, 16, 20, 32$ and 40.

Several properties of binary code waveforms are desirable if they are to be used in implementing pulse compression in the target sensor component of a missile or fire-control system. These include very low or zero cross correlation with other binary codes that may be implemented in sensors deployed nearby. These properties would ensure that there would be little or no degradation in sensor system performance due to out-of-range clutter returns, multiple target sidelobes, or from mutual interference between deployed sensors using different codes.

Long binary codes with the desired properties are required in order to implement waveforms with large time-bandwidth products and large pulse-width compression ratios. This document describes the structure and properties of such a waveform, called group complementary codes.

II. STRUCTURE AND PROPERTIES OF GROUP COMPLEMENTARY CODES

Group complementary codes are extensions of the complementary code concept introduced by Golay.[4] The codes discussed here are matrices of K by N binary elements, and the pulse compression processing involves transforming K long pulses, each coded with one of the K rows of N-bit binary words, into one single short pulse. Therefore, the pulse compression is a composite operation over a number of pulses rather than on a single pulse.

The implementation of the multipulse processing technique could take several forms but would necessarily require the storage of the partial correlation resulting from each of the K pulses to form the composite matrix correlation. One means of implementing the concept would utilize Charge Coupled Device (CCD) delay lines for storage of each of the K pulse correlations. The device would then provide K inputs for the formation of the composite compressed pulse. Another implementation would involve the use of one integrator to accumulate the range-time samples resulting from correlating the K pulses. Figure 4 illustrates the multiword pulse compression concept where τ_p is the width of each code pulse, T is the single pulse unambigious interval, and KT is the group unambigious interval.

A group complementary matrix is composed of K rows and N columns with each element being a pulse or minus "1". Each row is a code word used to encode each of K radio frequency pulses using bi-phase modulation.

The first K-1 rows are shifted versions of the same maximal length code word but with an extra bit of value "1" added at the end. The last row of the matrix is composed of all "1"s. Since a new matrix may be established for each unique maximal length word, M unique matrices exist for M unique code words.

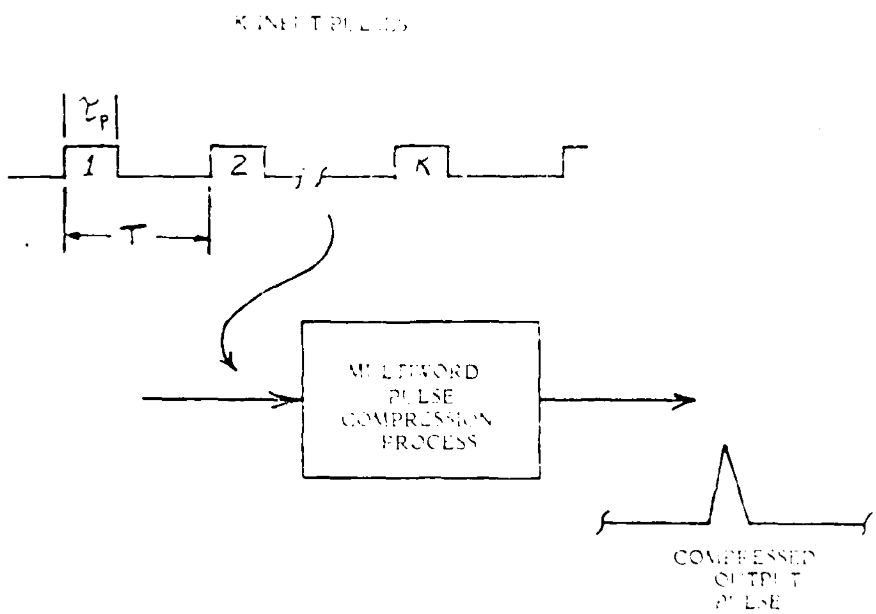


Figure 4. Multiword pulse compression.

A very large set of group complementary matrices may be generated from this configuration. An initial unique, but square, matrix may be operated upon in four different ways, in combination or separately, to generate new group complementary code matrices while maintaining the desired and beneficial properties: (1) one or more columns may be truncated, (2) columns or rows may be interchanged, (3) one or more rows may be complemented, and (4) one or more columns may be complemented. This provides a maximum number of possible code matr'ces as:

$$M \sum_{N=0}^{K-1} \frac{K!}{N!(K-N)!} \; N! \; K! \; 2^N \; 2^K ,$$

which reduces to

$$M(K!)^2 \; 2^K \sum_{N=1}^{K} \frac{2^N}{(K-N)!} \quad .$$

However this includes duplicate matrices and the total value only serves as a gross upper limit on the number of available matrices. For one unique maximal length code and an initial matrix with 16 rows and 15 columns this would result in an upper limit of more than 3.10×10^{36} group complementary code matrices as compared with 2^{NK} possible matrices without including truncated columns. This would be $2^{16(15)}$ or greater than 1.76×10^{72}.

Three examples of group complementary matrices are presented in Figure 5. The first is a square matrix with $N = K = 2^n$ rows and columns. Each of the first K-1 rows except for the last bit, is a shifted version of the same manimal length code. The last row is all "1"s. The second example is derived from the first by truncating the last column of the first martix and the third example is derived from the first by truncating the last two columns.

The first code structure shown in Figure 5 is an example of a square matrix, H, which satisfies

$$H \cdot H^T = NI$$

where H^T is the transpose of H, I is the identity matrix, and N is the order of H. Such a matrix is called a Hadamard matrix if its elements are +1's and -1's and satisfy the above relation.[5],[6] It is clear that a group complementary matrix with

$$K = N = 2^n$$

is a Hadamard matrix where n is an integer and is the order of the maximum length sequence comprizing the first N-1 elements of each row of the matrix. All other cases of group complementary matrices are truncated (columns truncated) Hadamard matrices.

Group complementary codes have optimized autocorrelation properties; that is, all autocorrelation sidelobes within the principal interpulse period are identically zero when the autocorrelation function is formed from the composite of K pulses. Figure 6 illustrates the autocorrelation of two repetitive waveforms, one using the same code word and the second using K code

Example 1: $K = N = 2^n$; n an integer

$$K = 8 \left\{ \overbrace{\begin{matrix} -1 & -1 & -1 & 1 & 1 & -1 & 1 & 1 \\ 1 & -1 & -1 & -1 & 1 & 1 & -1 & 1 \\ -1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 \\ -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 \\ -1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{matrix}}^{N = 8} \right.$$

Example 2: $N = K - 1 = 2^n - 1$; n an integer

$$K = 8 \left\{ \overbrace{\begin{matrix} -1 & -1 & -1 & 1 & 1 & -1 & 1 \\ 1 & -1 & -1 & -1 & 1 & 1 & -1 \\ -1 & 1 & -1 & -1 & -1 & 1 & 1 \\ 1 & -1 & 1 & -1 & -1 & -1 & 1 \\ 1 & 1 & -1 & 1 & -1 & -1 & -1 \\ -1 & 1 & 1 & -1 & 1 & -1 & -1 \\ -1 & -1 & 1 & 1 & -1 & 1 & -1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{matrix}}^{N = 7} \right.$$

Example 3: $N < K-1$

$$K = 8 \left\{ \overbrace{\begin{matrix} -1 & -1 & -1 & 1 & 1 & -1 \\ 1 & -1 & -1 & -1 & 1 & 1 \\ -1 & 1 & -1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 & -1 & -1 \\ 1 & 1 & -1 & 1 & -1 & -1 \\ -1 & 1 & 1 & -1 & 1 & -1 \\ -1 & -1 & 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{matrix}}^{N = 6} \right.$$

Figure 5. Group complementary code matrices: three examples.

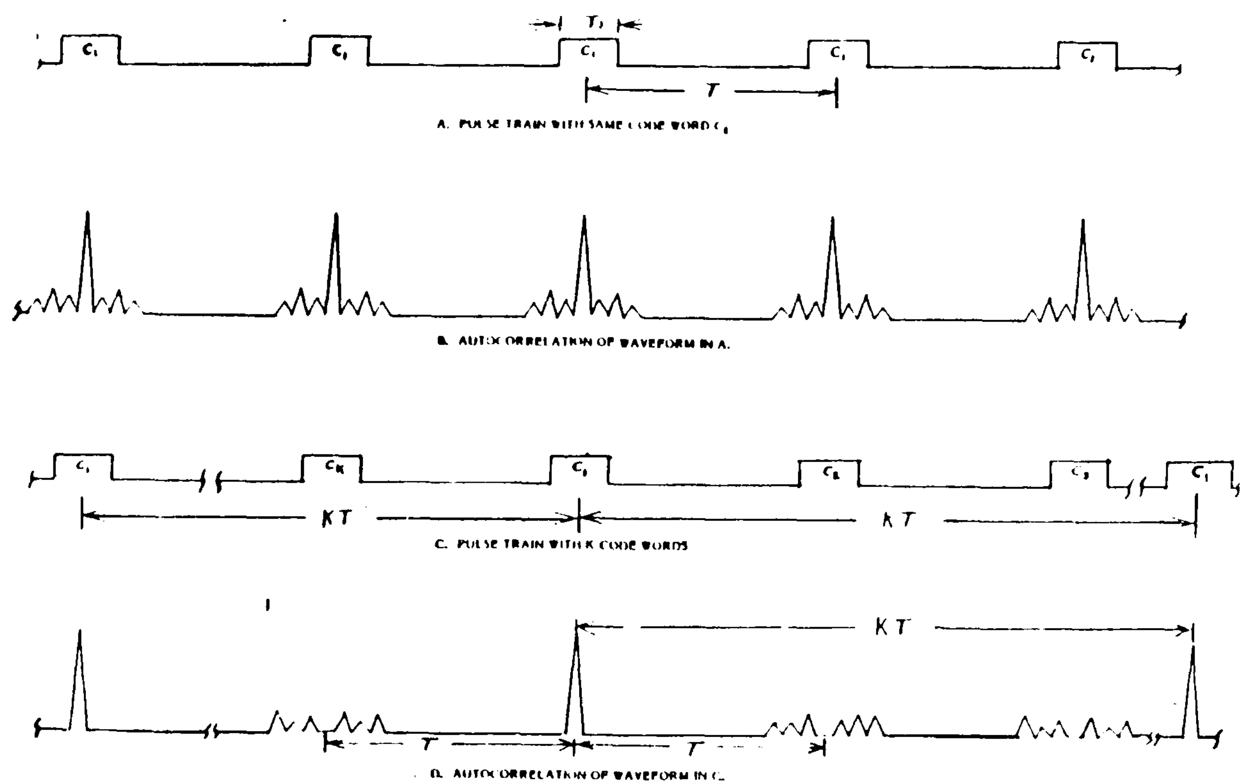


Figure 6. Autocorrelation of pulse train with one code word or group complementary code words.

words. With K code words, the principal unambigious interval has been increased from T to KT units and the sidelobes in the interval $\pm (T - T_i)$ have been reduced to zero. "Second-time-around" (ambigious) responses would be displaced from the peak of the autocorrelation function by a factor K as compared to the second-time-around reponses in an uncoded pulse sensor system. This is analogous to the case of interpulse coding where each pulse is modulated by a single binary element.

The structure of group complementary codes can be understood by considering each group complementary matrix as a composite of a number of vectors, each vector being a shifted version of a maximum length sequence, or a vector with all +1 elements. Figure 7 shows the structure of the group complementary matrix.

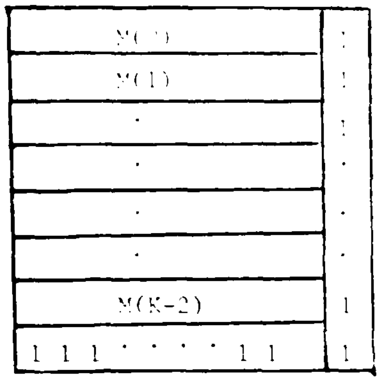


Figure 7. Structure of group complementary matrix as composite of maximal length vectors and all "1" vectors.

In the figure M(S) is a maximum length sequence with a cylic shift of S bins. For example, if

$$M(0) = -1 \ -1 \ -1 \ \ 1 \ \ 1 \ -1 \ \ 1$$
$$M(3) = \ \ 1 \ -1 \ \ 1 \ -1 \ -1 \ -1 \ \ 1.$$

The sturcture for calculating the correlation between a group complementary matrix and a shifted version of itself, at one particular shift, is indicated in Figure 8. The correlation for a given shift is the sum of K partial correlation results as shown in Figure 9, where $C_i(\cdot)$ is a partial correlation result, and it will be shown that

$$C(\tau) = \sum_{i=0}^{K-1} C_i(\cdot) = 0 \quad \text{for} \quad 1 \leq \tau \leq N-1$$

and

$$C(0) = \sum_{i=0}^{K-1} C_i(0) = KN.$$

The group complementary matrix can be segmented in a different but equivalent manner as shown in Figure 10 because each column is a previously identified maximal length code arranged in the order shown.

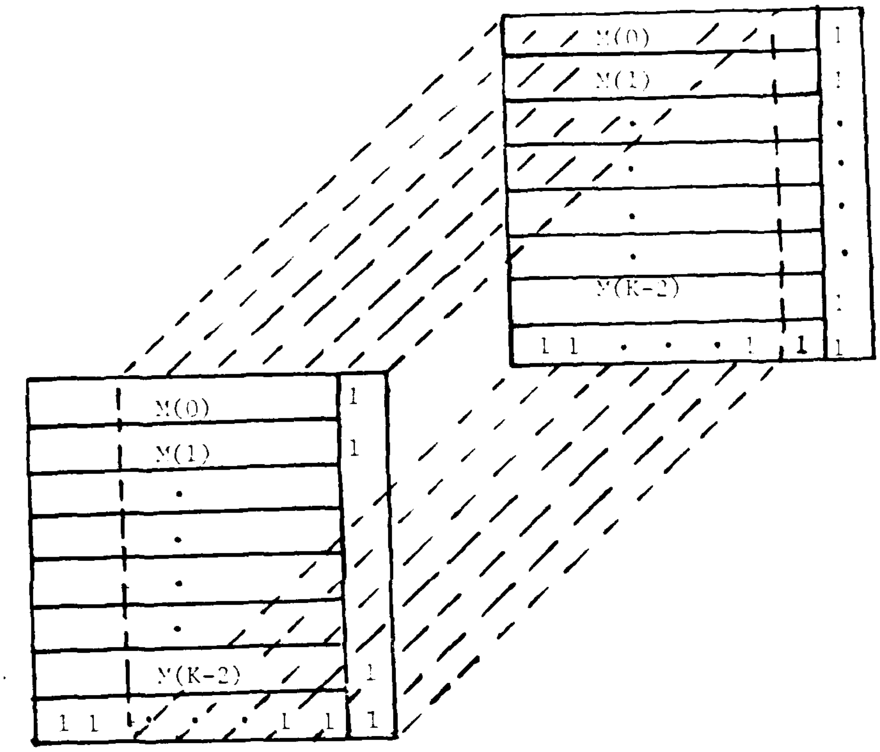


Figure 8. Formation of autocorrelation function for a group complementary matrix for a particular shift.

For this alternate interpretation of the matrix of Figure 10, the structure for calculation the correlation at a particular shift is shown in Figure 11. The correlation is the sum of $(N-\tau)$ partial correlation results as shown in Figure 12. From the two alternate interpretations of the matrix, with P_i defined by Figure 12,

$$C(\tau) = \sum_{i=0}^{K-1} C_i(\tau) = \sum_{i=1}^{N-\tau} P_i$$

because each product of elements contributing to the sum once and only once for row to row correlation exist once and only once in the sum for column to column correlation. In the latter there are fewer columns to add, but each column has more producted elements.

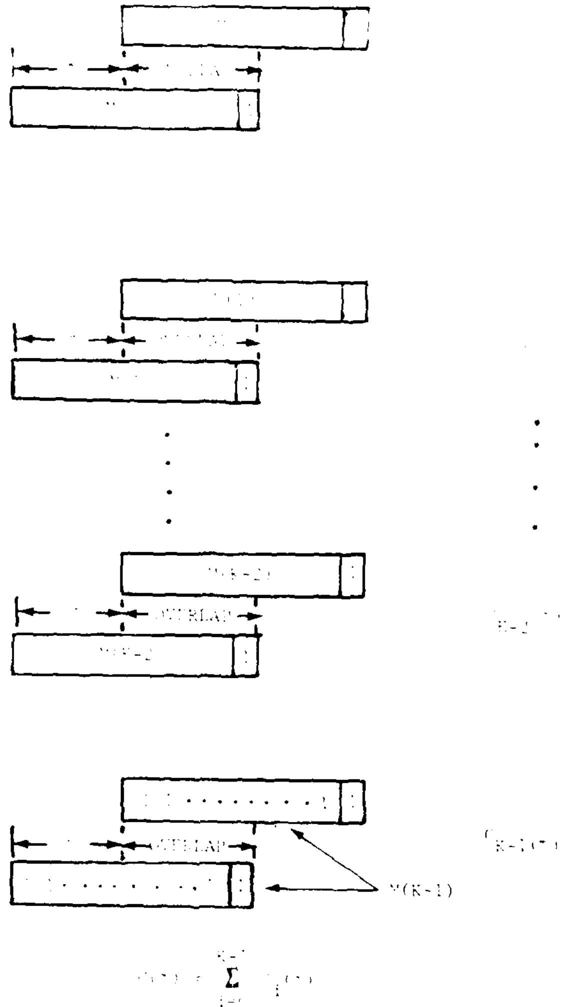


Figure 9. Autocorrelation for a shift for group complementary matrices.

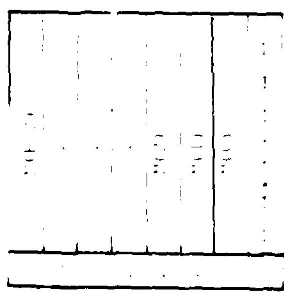


Figure 10. Alternate interpretation of the structure of group complementary matrix.

A detailed examination of the calculation of an P_i term shows that if $i \neq N - \tau$ the calculation is the correlation of a maximum length sequence with a shifted version of itself plus a 1. This result will always be 0. If $i = N - \tau$ the calculation is the correlation of a maximum length sequence plus a 1 with an all "1" vector. This result also will always be 0. And therefore

$$C(\tau) = \sum_{i=0}^{K-1} C_i(\tau) = \sum_{i-1}^{N-\tau} P_i = 0, \quad \tau \neq 0$$

since all Pi are zero. Also, $C(0) = KN$ since this is perfect correlation for no shift. An example of a group complementary matrix structure where eight code words are used, each with 6 bits, is shown in Figure 13 with the principal portion of its autocorrelation response.

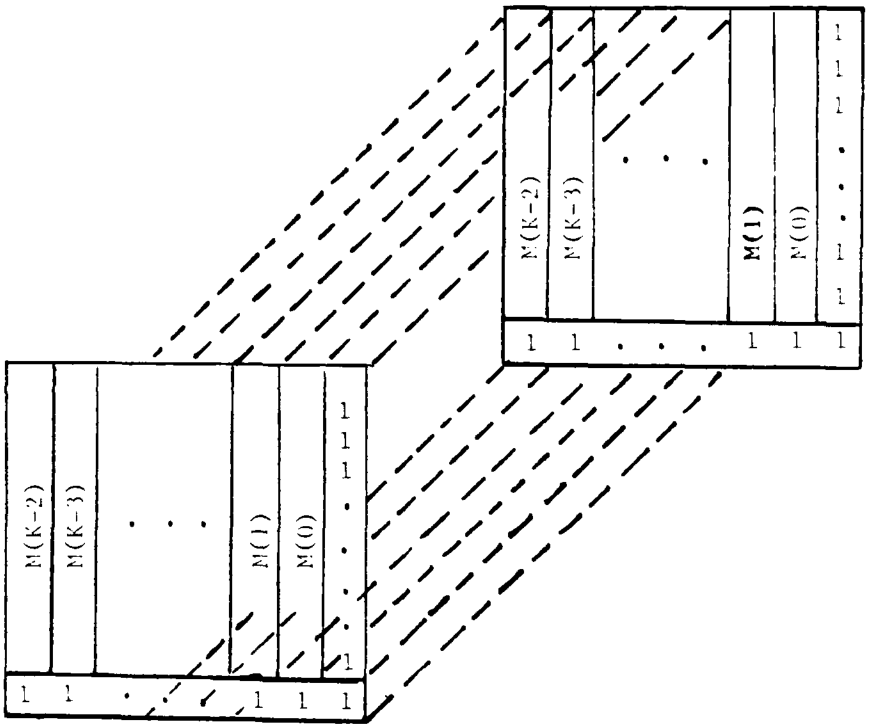


Figure 11. Formation of autocorrelation function for a group complementary martix.

Multiple-time-around responses are observed when the received code word occurs at multiples of the unambiguous interval. This results in weak responses which appear far removed from the main peak unambiguous response. Two cases may occur, one when different code words (rows) are in phase alignment and the second when partial alignment occurs.' In the first case the response is identically zero because two maximal length codes are cross correlated when one is a shifted version of the other and the extra bit at the end of each word reduces the correlation to zero. In the second case the response will be due to the aperiodic property of code word and can be minimized by choosing maximum length code words with manimum aperiodic sidelobes.

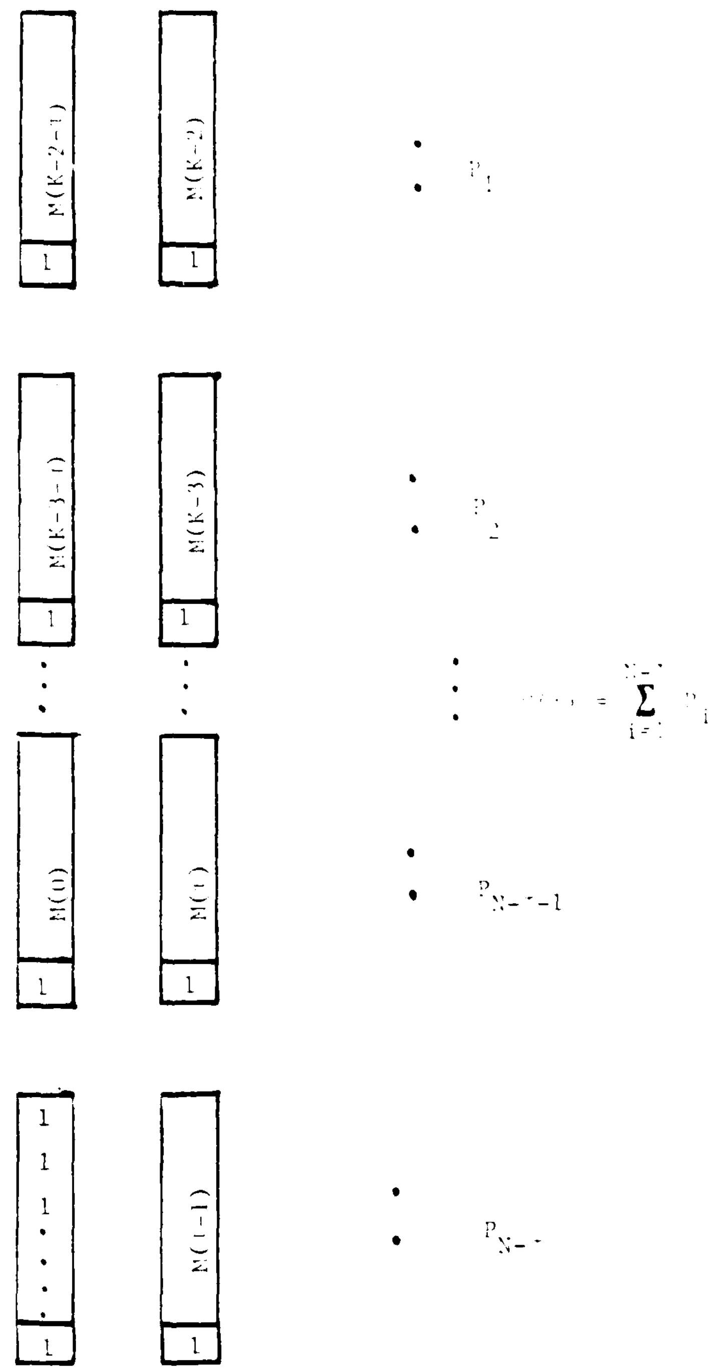


Figure 12. Alternate means of calculating autocorrelation shift for group complementary matrices.

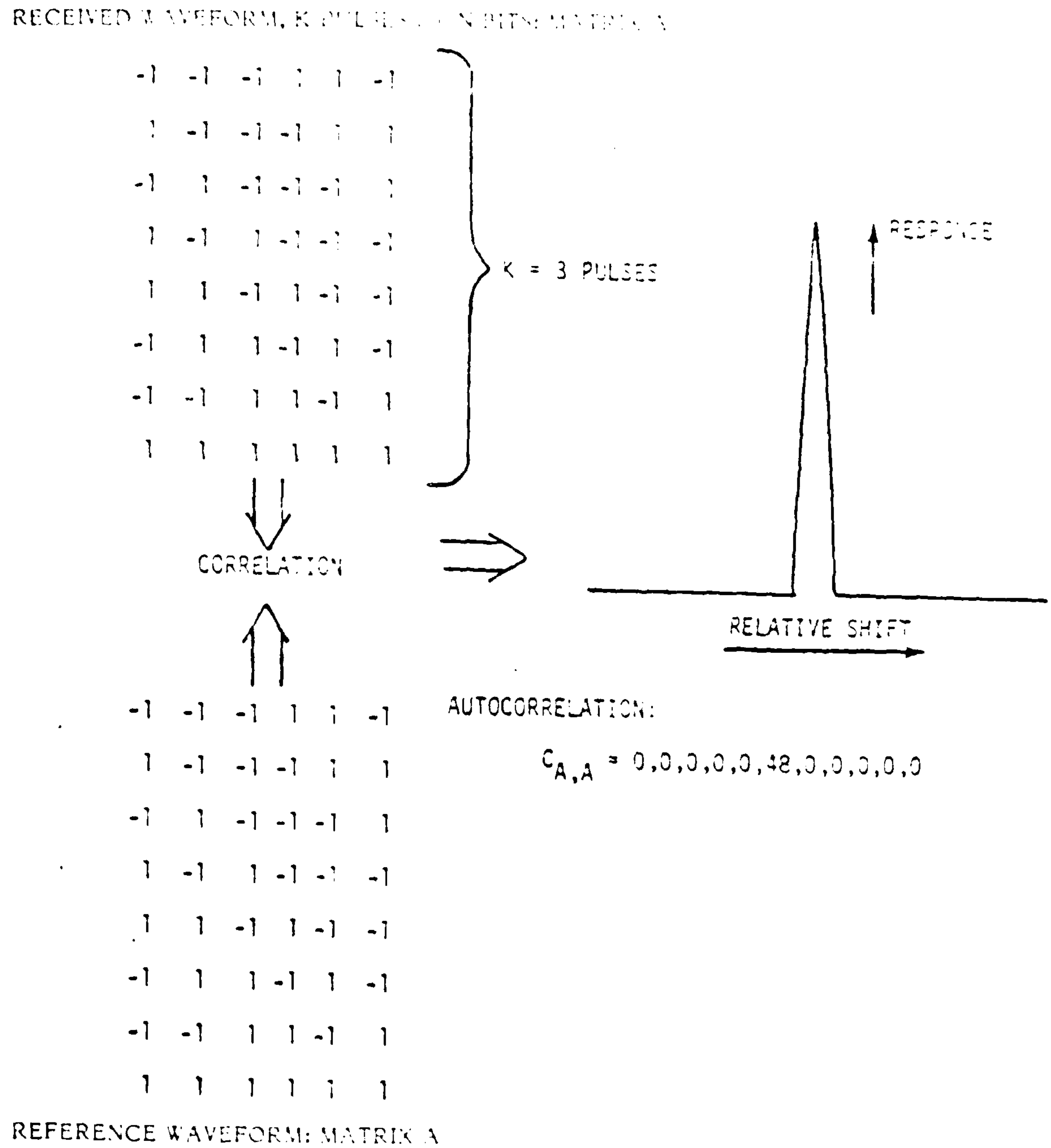


Figure 13. Pulse compression using group complementary binary codes with optimized autocorrelation properties.

III. GROUP COMPLEMENTARY CODE IMPLEMENTATION

Group complementary codes may be implemented in hardware in a number of ways. On particular implementation is shown in the simplified schematic diagram (Figure 13A). K pulses are generated and transmitted at a given pulse repetition rate (PRF). Each pulse is encoded with N bits of bi-phase modulation. After transit delay to the RF reflector of interest, the received signal is cross correlated with an appropriately delay reference code. K pulses are added to form this range gate output. This process effectively accomplishes range gating, and the desired number of range gates are formed by a corresponding number of correlators.

The RF pulses to be transmitted are generated in the Transmitter/Local Oscillator Frequency Reference Unit. Each pulse is encoded using bi-phase modulation and this is accomplished at the Modulator where the phase during the RF pulse is changed by 180 degrees or not changed according to the base band video code word. The code word is developed in the Code Generator Unit and it is composed of N bits of a digital word which controls the phase changes according to the bit pattern. The code word is commensurate with the RF pulse in time of occurrence and duration.

The encoded RF pulses are routed to the circulator which in turn directs the RF energy to the antenna. The radiated pulses are received by the antenna upon reflection from objects in the antenna field of view. The received pulses are routed through the circulator to the first mixer. Here the received signals are translated in frequency to the first Intermediate Frequency (IF) amplifier's center frequency for amplification and filtering. Output of this unit is routed to the second mixer for further translation and amplification at the second IF frequency. The output of the second IF amplifier becomes one of the two input signals which the correlator operates upon. The second input signal to the correlator is a base band code word derived in the Code Generator Unit. The correlator is composed of a mixer and an integrator to carry out the cross correlation function between the received code word and the reference code word. The reference code word is a delayed version of the transmitted code word. The delay corresponds to the range of interest for a given range gate. Additional range gates are formed with additional correlators and delayed reference code words. The range gate is formed by summing the output of the correlator after each of K pulses are received. This summation is accomplished by the integrator and its output becomes the range gate output.

The Code Generator Unit develops the code words to be transmitted. K unique code words are transmitted before the sequence is repeated. Each correlator requires a reference code word of appropriate delay for each transmitted code word. The reference code words are also developed in the Code Generator Unit. The code words are stored in the Code Storage Unit, a read only memory (ROM). Each word is N bits in length and at the appropriate time is transferred to the Transmit Code Register or the Reference Code Register. This is accomplished by the Timing and Control Unit which feeds the code word address generator. Each code word is stored at a unique address in the ROM and as each word is addressed it is transferred to the parallel-in, serial out code register for either transmission or reference. Each transmit word is shifted out of the register in serial form to encode the RF pulse while the same word is shifted

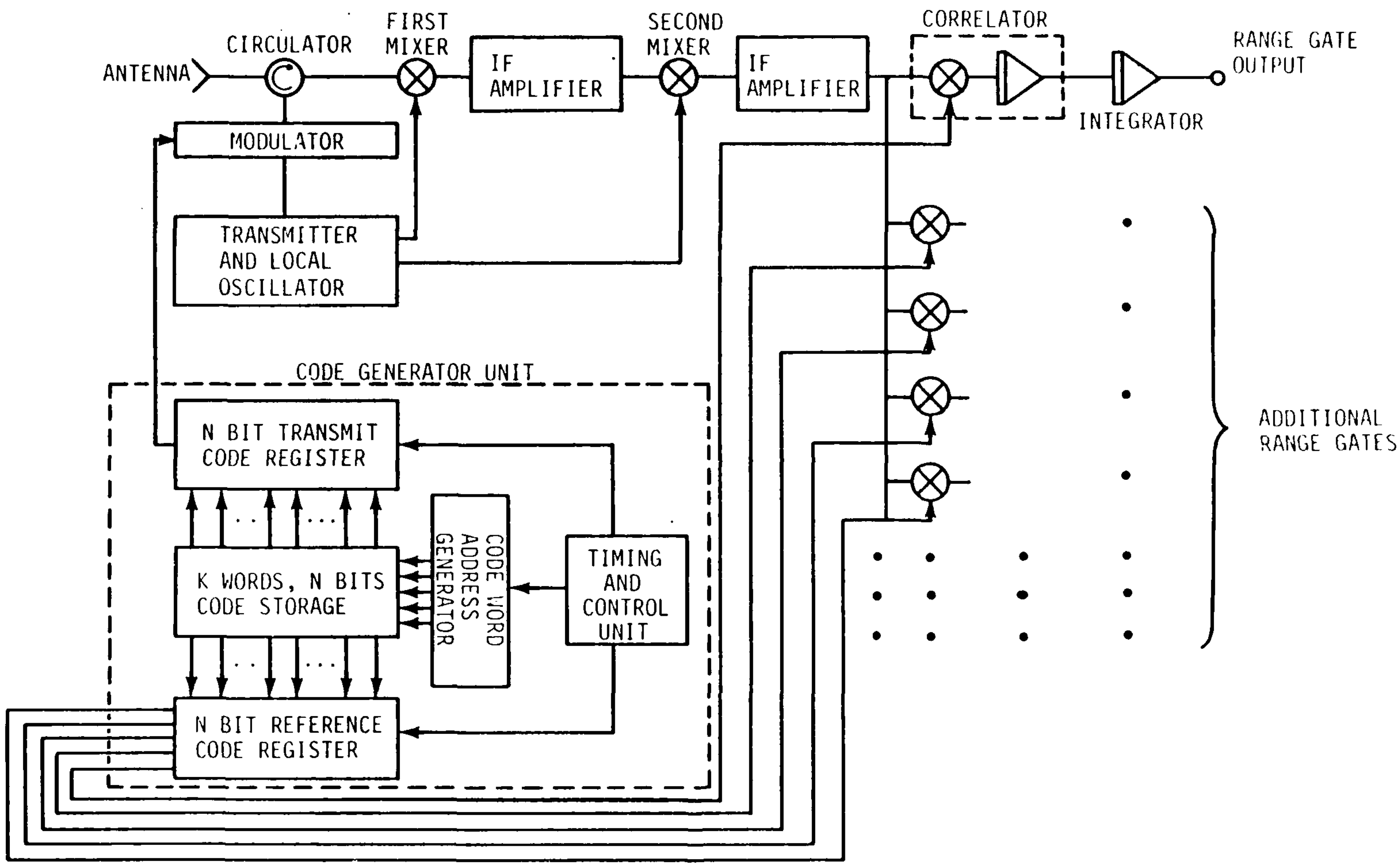


Figure 13A. Simplified diagram of group complementary code inplementation.

in serial form out of the Code Reference Register at the correct time for a given correlator; then a one bit delayed version is shifted out on a separate line to the next correlator. The proper reference code is routed in the same fashion, with the proper shift and delay, to all other correlators. The process is then repeated with a new code word on the next transmission, with the Timing and Control Unit providing the proper signals to maintain proper synchronization.

IV. CODE OTRHOGONALITY

Group complementary codes have another beneficial property which can be exploited in sensor design and developments. This feature involves mutual orthogonality of code matrixes. As previously discussed, the group complementary code matrix, A, from Figure 13 may be operated upon to create new matrices while maintaining the autocorrelation properties of the original matrix. A special case is when N is even and N/2 columns of the original matrix are inverted to form a second matrix. For this case, the cross correlation between the two matrices is identically zero. This is an ideal property for two closely deployed sensors, whose transmissions can be sychronized, each using one of the code matrices for pulse compression. This provides mutually noninterferring operation over the unambigious interval T. Some sidelobes will be formed at time shifts greater than T but will be exactly zero again at multiples of T. Figure 14 shows an example of two mutually orthogonal matrices where the second, fourth and sixth columns of the first matrix have been inverted.

From a given K by N group complementary "seed" matrix, where N is even, a new set of N code matrices can be formed with optimized autocorrelation and mutual noninterference (orthogonal) properties. A synthesis procedure can be formulated as a matrix operation,

$$A_i = A_o \, \underline{T_i} \, \underline{K}$$

where A_o is the "seed" code matrix, $\underline{T_i}$ is an N element vector with binary elements +1 and -1, and $\underline{K}$ is an N element vector with all ones,

$$\underline{K} = (1\ 1\ 1\ \ldots.1).$$

$\{\underline{T}_i\}$ is a set of vectors of order N. When comparing element for element between any two vectors in the set $\{T_i\}$, there are exactly N/2 agreements and N/2 disagreements for an 8 x 8 seed code matrix.

A set of $\underline{T_i}$ vectors are shown in Figure 15 and these would permit generation of eight mutually orthogonal matrices from the initial seed A_o. If M orthogonal matrices are desired (M equal to or less than N) M-$\underline{T}_i$ vectors are required.

RECEIVED WAVEFORM FROM SENSOR A BY SENSOR B

$$\left.\begin{matrix} -1 & -1 & -1 & 1 & 1 & -1 \\ 1 & -1 & -1 & -1 & 1 & 1 \\ -1 & 1 & -1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 & -1 & -1 \\ 1 & 1 & -1 & 1 & -1 & -1 \\ -1 & 1 & 1 & -1 & 1 & -1 \\ -1 & -1 & 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{matrix}\right\} A$$

$\Downarrow$

CORRELATION $\Rightarrow$ Identically zero

Cross-correlation

$T_1 - T \leq \tau \leq T - T_1$

$\Uparrow$

$$\left.\begin{matrix} -1 & 1 & -1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 & 1 & -1 \\ -1 & -1 & -1 & 1 & -1 & -1 \\ 1 & 1 & 1 & 1 & -1 & 1 \\ 1 & -1 & -1 & -1 & -1 & 1 \\ -1 & -1 & 1 & 1 & 1 & 1 \\ -1 & 1 & 1 & -1 & -1 & -1 \\ 1 & -1 & 1 & -1 & 1 & -1 \end{matrix}\right\} B$$

REFERENCE WAVEFORM

Figure 14. Example of mutually orthogonal martices.

$$\underline{I}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ -1 \\ -1 \\ -1 \\ -1 \end{bmatrix} \quad \underline{I}_2 = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \\ 1 \\ 1 \\ -1 \\ -1 \end{bmatrix} \quad \underline{I}_3 = \begin{bmatrix} -1 \\ 1 \\ 1 \\ -1 \\ 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}$$

$$\underline{I}_4 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \\ 1 \\ -1 \\ 1 \\ -1 \end{bmatrix} \quad \underline{I}_5 = \begin{bmatrix} -1 \\ -1 \\ 1 \\ 1 \\ 1 \\ 1 \\ -1 \\ -1 \end{bmatrix} \quad \underline{I}_6 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \\ -1 \\ 1 \\ -1 \\ 1 \end{bmatrix}$$

$$\underline{I}_7 = \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \\ 1 \\ -1 \\ -1 \\ 1 \end{bmatrix} \quad \underline{I}_8 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

Figure 15. Eight transformation vectors, which may operate on a seed 8 by 8 group complementary code matrix and produce a set of eight mutually orthogonal complementary code matrices with optimized autocorrelation properties.

The set of transformation vectors shown is not the only set that will synthesize a set of orthogonal group complementary code matrices from the seed, but it serves as an example of the procedure. Specifically each vector is orthogonal to all other vectors in the set.

The number of mutually orthogonal, N matrix, sets that may be formed using a set of transformation vectors can be large. For example, for $N = 4$, 16 $\underline{T}_i$ vectors are available. Figure 16 presents each vector which accounts for all 16 possible cases since N is equal to 4. Eight pairs of these vectors are complements of each other and they are not orthogonal. Therefore, the pair members may not be used together. Thirty-two sets of 4 orthogonal matrices may be generated using selected members of the vectors while excluding one pair member.

Figure 17 presents the selection of transformation vectors, in groups of four, which may be used to generate 32 sets of matrices in which each of the four members are orthogonal among themselves.

The existence of sets of orthogonal group complementary matrices allows the synthesis of a large group complementary matrix structure with N columns and K rows and results in the condition $N>K$.

The synthesis procedure is to form a composite matrix by horizontal concatenation of all the members, or a subset of the members, of a set of mutually orthogonal group complementary matrices. For example, a set of four mutually orthogonal group complementary matrices are:

$$A_0 = \begin{bmatrix} -1 & -1 & 1 & 1 \\ -1 & 1 & -1 & 1 \\ 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 \\ -1 & 1 & -1 & 1 \\ -1 & -1 & 1 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 \\ -1 & -1 & 1 & 1 \\ -1 & 1 & -1 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ -1 & -1 & -1 & -1 \\ -1 & 1 & 1 & -1 \end{bmatrix}$$

A concatenated matrix, $A_0\ A_1\ A_2\ A_3$ [illegible]

$$\begin{bmatrix} -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 \end{bmatrix}$$

$$\underline{I}_1 = \begin{bmatrix} -1 \\ -1 \\ -1 \\ -1 \end{bmatrix} \quad \underline{I}_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \quad \underline{I}_3 = \begin{bmatrix} -1 \\ -1 \\ 1 \\ 1 \end{bmatrix} \quad \underline{I}_4 = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$$

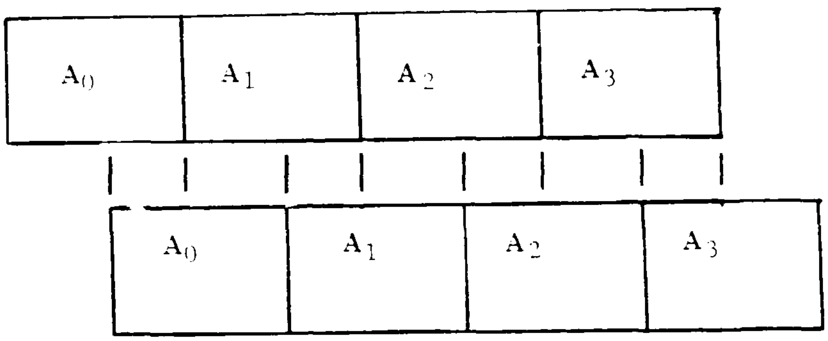
$$\underline{I}_5 = \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix} \quad \underline{I}_6 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix} \quad \underline{I}_7 = \begin{bmatrix} -1 \\ 1 \\ 1 \\ -1 \end{bmatrix} \quad \underline{I}_8 = \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}$$

$$\underline{I}_9 = \begin{bmatrix} -1 \\ -1 \\ -1 \\ 1 \end{bmatrix} \quad \underline{I}_{10} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \end{bmatrix} \quad \underline{I}_{11} = \begin{bmatrix} -1 \\ -1 \\ 1 \\ -1 \end{bmatrix} \quad \underline{I}_{12} = \begin{bmatrix} 1 \\ 1 \\ -1 \\ 1 \end{bmatrix}$$

$$\underline{I}_{13} = \begin{bmatrix} -1 \\ 1 \\ -1 \\ -1 \end{bmatrix} \quad \underline{I}_{14} = \begin{bmatrix} 1 \\ -1 \\ 1 \\ 1 \end{bmatrix} \quad \underline{I}_{15} = \begin{bmatrix} 1 \\ -1 \\ -1 \\ -1 \end{bmatrix} \quad \underline{I}_{16} = \begin{bmatrix} -1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

Figure 16. Sixteen vectors which permit generation of 32 sets of matrices, each set has four mutually orthogonal members.

which is in a 16 column by 4 row matrix. That this matrix has group complementary properties can be shown by observing the overlap regions in the diagram below:



The total autocorrelation is the sum of several partial correlations, all of which are zero from the mutually orthogonal and zero sidelobe matrix property. The new, but larger matrix retains the desired group complementary properties.

1. T_1, T_3, T_5, T_7
2. T_2, T_3, T_5, T_7
3. T_1, T_4, T_5, T_7
4. T_2, T_4, T_5, T_7
5. T_1, T_3, T_6, T_7
6. T_2, T_3, T_6, T_7
7. T_1, T_4, T_6, T_7
8. T_2, T_4, T_6, T_7
9. T_1, T_3, T_5, T_8
10. T_2, T_3, T_5, T_8
11. T_1, T_4, T_5, T_8
12. T_2, T_4, T_5, T_8
13. T_1, T_3, T_6, T_8
14. T_2, T_3, T_6, T_8
15. T_1, T_4, T_6, T_8
16. T_2, T_4, T_6, T_8
17. T_9, T_{11}, T_{13}, T_{15}
18. T_{10}, T_{11}, T_{13}, T_{15}
19. T_9, T_{12}, T_{13}, T_{15}
20. T_{10}, T_{12}, T_{13}, T_{15}
21. T_9, T_{11}, T_{14}, T_{15}
22. T_{10}, T_{11}, T_{14}, T_{15}
23. T_9, T_{12}, T_{14}, T_{15}
24. T_{10}, T_{12}, T_{14}, T_{15}
25. T_9, T_{11}, T_{13}, T_{16}
26. T_{10}, T_{11}, T_{13}, T_{16}
27. T_9, T_{12}, T_{13}, T_{16}
28. T_{10}, T_{12}, T_{13}, T_{16}
29. T_9, T_{11}, T_{14}, T_{16}
30. T_{10}, T_{11}, T_{14}, T_{16}
31. T_9, T_{12}, T_{14}, T_{16}
32. T_{10}, T_{12}, T_{14}, T_{16}

Figure 17. Vectors for generating thirty-two sets of mutually orthogonal matrices with four members each.

IV. SUMMARY

Group complementary code sets have optimized autocorrelation and cross-correlation properties over the single pulse unambiguous interval. These properties, coupled with the relative ease of implementing biphase binary coding, make these waveforms strong candidates for sensor pulse-compression applications. The advantages of group complementary code sets are summarized below:

- Matched Filtering for Optimized Detection
- Reduced sensitivity to out-of-range clutter and multi-target returns through optimized autocorrelation.
- Large number of Group Complementary Codes available.
- Reduced sensitivity to mutual interference through orthogonal code sets.
- Large sets of orthogonal waveforms available.
- Ease of implementing the waveforms in a sensor system with read-only memory (ROM) for storage of codes and biphase carrier modulation.

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